

centre of the motion (*a*), be so chosen, that the result obtained is greater than if any other point in the intrados were chosen for a centre of motion, it is the area sought for, and (*a*) is the point of fracture. If the whole thickness of the arch and load at the crown (*c g*) is called *T*,

$$S = T \cdot R^2 \frac{\sin^2 \theta}{2} + R^3 M$$

$$\frac{R \cdot \text{vers.} \theta + K}{2}$$

$$T \cdot \frac{\sin^2 \theta}{2} + R \cdot M$$

$$S = R^2 \frac{R \cdot \text{vers.} \theta + K}{2} \quad (1)$$

If the arch be loaded as represented by Fig. 2, the fall of the load (*a b*) over the springing being called *F*,

$$T \cdot \frac{\sin^2 \theta}{2} + R \cdot M = F \cdot \frac{\sin^2 \theta}{2}$$

$$S = R^2 \frac{R \cdot \text{vers.} \theta + K}{2} \quad (2)$$

If the arch be loaded as represented by Fig. 3, the roadway being parabolic, and the ordinate of the parabola (*a b*) over the springing called *F*,

$$T \cdot \frac{\sin^2 \theta}{2} + R \cdot M = F \cdot \frac{\sin^2 \theta}{2}$$

$$S = R^2 \frac{R \cdot \text{vers.} \theta + K}{2} \quad (3)$$

to find the values of *S*, for elliptical arches loaded in a corresponding manner to the above (compare Fig. 1 with Fig. 4), for *RM* and *R* versed sine in the formula 1, 2, and 3, substitute *HM* and *H* versed sine. In the formula so altered, *R* is the semi-span, and *H* the rise. Thus the rule for finding *S* for the bridge, represented by Fig. 4 is,

$$T \cdot \frac{\sin^2 \theta}{2} + HM$$

$$S = R^2 \frac{H \cdot \text{vers.} \theta + K}{2} \quad (4)$$

In the absence of tables, or when the formulae are intricate, the simplest way, perhaps, to make these calculations is to divide the area of the section into triangles by equidistant parallel vertical lines and diagonals, and to compute the contents and moments of the different triangles as though they were rectilinear; thus the area (*abcd*) Fig. 5 is nearly equal to

$$1 \times \left( \frac{e}{2} + f + g + h + \frac{i}{2} \right) \quad (5)$$

Its moment nearly equal to

$$R \times \left( \frac{e}{6} + f + 2g + 3h + \frac{3i}{2} + \frac{j}{3} \right) \quad (6)$$

and the distance of its centre of gravity from (*ab*) is nearly equal to the second of these two formulae divided by the first. If the point in the intrados (*a*) first assumed be not near the point of fracture, let some other point, as (*k*), be assumed (above or below it, as the case may require), and let the moments of the two dotted triangles (*abk*) and (*jkb*) be added to (or deducted from, if necessary) the moment of (*abcd*) with regard to the point (*k*). By varying this point and using the moment so obtained, the horizontal stress may be arrived at quite near enough for practical purposes.

To find an ordinate to the line of equilibrium, divide the moment of any part of the section, as (*afgc*) or (*chij*), fig. 1, by *S*. Thus, the rule for finding the ordinate (*kl*) equal to *Y*, (*ch*) being called (*i*), is,

$$Y = \frac{\text{Moment of } (chij)}{S}$$

$$R^2 \cdot \left( \frac{T \cdot \sin^2 \theta}{2} + RM \right) = \left( \frac{T \cdot \sin^2 \theta}{2} + RM \right)$$

$$R^2 \cdot \left( \frac{R \cdot \text{vers.} \theta + K}{2} \right) = \left( \frac{S}{R^2} \right) \quad (7)$$

To find the linear or columnal measure of the horizontal stress divide *S* by *K*.

$$C = \frac{S}{K} \quad (8)$$

To find the tangent to the curve of equilibrium at any point, as (*a*), fig. 4. Upon a horizontal line drawn from the point, set off with any convenient scale (*ab*) equal to *S*, and on a perpendicular line through (*b*), (*cb*) equal to the area (*adef*), the line between (*a*) and (*c*) is the tangent. When the tangent of the curve of equilibrium, at any point, agrees

with the tangent of the intrados of an arch, at that point, it is a point of fracture.

**Examples.**—Required *S* the area proportional to the horizontal stress of a semicircular arch, loaded as shown by Fig. 1. *R* = 30 feet, *T* = 7.7121, *K* = 3 feet. By the first formula, assuming the point of fracture to be at 54 degrees,

$$S = 30 \times 30 \times 7.7121 \times \frac{.65451}{2} + 30 \times .0193 =$$

$$30 \times .41221 + \frac{3}{2}$$

201.375 feet.

By a similar operation the value of *S* to 53 degrees was found to be 201.333, and to 54 degrees 201.333. The point of fracture is very nearly at 54 degrees; the thickness at the crown having been previously calculated that it might be so. The trifling difference between these results shows that an error of a degree or two in determining the position of the points of fracture is of no importance. This will appear more evident from the linear or columnal measure of the horizontal stress deduced from the above approximations to *S*, (5th) Rule.

$$C = \frac{201.375}{3} = 67.125$$

$$C = \frac{201.333}{3} = 67.111$$

$$C = \frac{201.339}{3} = 67.113$$

The greatest difference between these is only a sixth of an inch.

Required the ordinates (*kl*, *mn*, *op*) of the line of equilibrium, corresponding to 30, 54, and 90 degrees of the intrados: the depth of the permanent load and arch (*t*), at the crown, being 4.5 feet. By the seventh formula,

$$Y \text{ to } 30^\circ = 4.5 \times .25 + 30 \times .00267$$

$$\left( \frac{7.7121 \times .65451 + 30 \times .0193}{30 \times .41221 + \frac{3}{2}} \right) =$$

$$\frac{.6246}{.22375} = 2.787 \text{ feet.}$$

$$Y \text{ to } 54^\circ = \frac{2.0516}{.22375} = 9.169 \text{ feet.}$$

$$Y \text{ to } 90^\circ = \frac{3.6882}{.22375} = 16.483 \text{ feet.}$$

Required *S* for an elliptical arch loaded as represented by Fig. 4, the semi-span *R* = 45 feet, the Rise *H* = 30 feet, *T* = 7.7121, *K* = 3 feet.

All the quantities forming the fractional part of the fourth formula are the same as in the previous example, the only difference is in the squares of the semi-spans.

$$30 : 45 :: 1 : 2.25$$

$$S = 201.375 \times 2.25 = 453.093 \text{ feet.}$$

$$C = 67.125 \times 2.25 = 151.031 \text{ feet.}$$

The ordinates (*gh*), (*ij*), (*kl*), are the same as those previously determined. The points of fracture have very nearly the position of those of the bridge of Nogent-sur-Seine, determined from observation by Peyronnet.

\* M. Gardel has calculated the points of fracture and thrusts of various semicircular bridges, upon principles somewhat different from those here adopted—the points of fracture to a hundredth part of a degree. M. Gardel's thrust is the pressure which one part of an arch, with its load, will exert to overthrow the remaining part and the abutment wall, if the curve of equilibrium be supposed to touch the extrados of the arch at the crown, and the intrados at two points. The thrust so estimated does not appear to the writer to be the true stress or pressure upon the materials. The resistance is in every particle of the vertical section of the key, and the resultant resistance of the particles, if the section be rectangular, is at the middle of the depth; with this resultant, according to the principles first explained by Galileo in his dialogues on the resistance of solid bodies, the stress should be equated, otherwise the resistance of the materials must be supposed to exist in the upper edge of the arch alone, and consequently to be enormous, which is far from being the case; on the contrary, it does not require any great force to compress the hardest materials used in building—a slight tap will make a sound stone ring like a bell, and when vibration occurs, it is certain that there is an alternate compression and dilatation of the particles acted upon; a carriage passing over an arch will set it in motion, which must necessarily cause the full momentum of one part of it to act against its opposite part, and that to react in its turn, not through immovable points or lines, but through the natural masses. M. Gardel's method, applied to determine the horizontal stress of a plinth, will give less than half of that determined by the other. According to his tables, an arch with level roadway of 50 feet span, 18 inches thick, and the whole depth at the vertex 7 feet 6 inches, will have the point of fracture at 54.01 degrees, and the horizontal thrust proportional to  $200 \pi \cdot 21.792 = 195.938$  feet, which does not differ much from the ratio of the stress determined above for an arch of the same span 3 feet thick, with the loading rather greater.

If *R* be 60 feet, and the vertical dimensions be as before—

$$30 : 60 :: 1 : 2 \quad 30^\circ : 60^\circ :: 1 : 4$$

$$S = 201.375 \times 4 = 805.5$$

$$C = 67.125 \times 4 = 268.5$$

If the span of a semi-elliptical bridge be the same as that of a semi-circular one, and the corresponding vertical dimensions have the same ratio, *S* is always the same, and the columnal measures are inversely as the depths of the keys. J. P. W.

#### NOTES IN THE PROVINCES.

THE two new ordnance stores in Portsea are to be built close to the ramparts, and an artesian well is to be sunk in one of them. The buildings, it is said, will be commenced immediately.—The contribution of 200*l.* by Miss Littlehales, for a stained-glass window in St. Lawrence's Church, Winchester, has produced a strong inclination to realise, by rate, a further sum of 400*l.* to 500*l.*, in order to "heft the church for such a window."—The Bromsgrove vestry have under consideration plans and specifications, by Messrs. Rowlands and Day, of Worcester, architects, for the repairs of the church, at a cost of about 2,000*l.*—The first stone of the Southgate-street dock, at Gloucester basin, was laid on Tuesday week before last. The dock will be opened for business, it is thought, in March next.—The ancient houses in High-street, Old Cheltenham, opposite Cambray-place, supposed to have been in existence three centuries, have been levelled to the ground, to furnish a site for better buildings.—On Thursday week the gate of the new lock at Cumberland basin, Bristol, nearest the river, was hung. It is said to be quite a novelty. The span is 58 feet; depth, 29 feet; breadth at lower part, 12 feet, decreasing in a curved line to 3 feet. It is composed of many wrought iron plates, upon a frame of angle iron in three decks, forming water-tight compartments, and adding to its strength. The sluices are worked by right and left hand screws, and counter-balance each other. It opens as easily as an ordinary lock gate. There is a face of mahogany on the gate, which, when shut, bears upon a face of masonry, rendering the whole tight. The entire weight is about 95 tons. It was built at the Great Western Steamship works, under the superintendence of Mr. Jones.—The new entrance to Aberystwyth is progressing; the foundation-stones of the bridge over the river Keny having been laid on Monday week. The contractor is Mr. Prosser. The new line of road from the bridge to its junction with Mill-street, forms an *S* curve. The works are superintended by Messrs. Sayce and Price, C.E., and will probably be completed by the end of the year.—A movement is to be commenced at Birmingham for the erection of model lodging-houses.—The Roman Catholic Church of St. Chad, in Bath-street, Birmingham, has been lately decorated under the superintendence of Mr. John Hardman, of that town, who, it is understood, has borne the whole expense. The roofs of the aisles are painted in a style corresponding to that of the nave; the decorations of the choir are specially noted. The screen and the rood have been cleaned, and the roof has been coloured. The beams springing from the arches are gilt, and the roof compartmented by gilded beams, each compartment coloured blue, and powdered with stars, with the emblem of the cherubim, set in a quatre-foil, in the centre of each. A double lancet decorated window has been introduced into the choir.—The vestry of St. Paul's, Bedford, have resolved to raise 1,500*l.* for repairs to the church generally, and particularly to the roof, which, according to the architects employed, Messrs. Wing and Jackson, stands much in need of it.—The first stone of the new church and school at Limalade, according to the *Northampton Herald*, was laid on Wednesday week.—The Rounford vestry have resolved on erecting their proposed new church on the site of the present one, *alias* St. Edward's Chapel.—The new Corn Exchange at Lincoln, according to the local *Times*, is progressing rapidly towards completion. The clock turret is to cost about 1,000*l.*—The Victoria Promenade and Polytechnic Rooms, Bridlington Quay, have been thrown